Parity Doubling in the Meson Spectrum

Eric S. Swanson[1,2]

[1] *Department of Physics and Astronomy, University of Pittsburgh, Pittsburgh PA 15260*
[2] *Jefferson Lab, 12000 Jefferson Ave, Newport News, VA 23606*

A simple argument for the restoration of parity symmetry high in the hadron spectrum is presented. The restoration scale is estimated to be 2.5 GeV. This in turn implies that typical quark model phenomenology such as scalar confinement or the $^3P_0$ decay model are only useful for low lying states. Minimal requirements for constructing more general phenomenologies are discussed. An additional mass degeneracy between J^{++} and J^{--} states is shown to occur and an isovector 3^{++} state is predicted at roughly 1700 MeV, in contradiction with the naive quark model. Similarly, isovector and isoscalar 4^{--} states are predicted at 2000 MeV. Finally, these results imply that Regge trajectories must become nonlinear at high spin.

I. INTRODUCTION

Spontaneous chiral symmetry breaking is one of the celebrated phenomena of low energy QCD. Its immediate consequence is a triplet (or octet) of light mesons whose existence is of central importance to much of nuclear and hadronic physics. Here I address the issue of the restoration of this symmetry (or rather, the irrelevance of its breaking) high in the hadron spectrum. A simple argument in support of this notion is presented and some implications are discussed. An important conclusion is that the nonrelativistic quark model must fail at some mass scale, and this scale can be surprisingly low.

The standard proof that chiral pions are massless supposes that the action and measure of QCD are invariant under a continuous chiral transformation. This implies that the second derivative of the effective potential is zero at the classical field minimum which in turn implies that the connected one-particle-irreducible propagator has a pole at zero momentum. The number of such poles is equal to the number of independent broken symmetries. Thus only a single triplet of pions is predicted and Goldstone's theorem has nothing to say about the excited pion spectrum. Indeed, the relatively close masses of the π' (1300 MeV) and the ρ' (1450 MeV) is a strong indication that the π' may be considered an ordinary $q\bar{q}$ bound state. It is thus tempting to speculate that chiral symmetry is irrelevant high in the hadron spectrum. Certainly, one expects this to be the case once the typical momentum transfers in hadronic bound states exceed the chiral symmetry breaking scale, $\langle p\rangle >> \Lambda_{\chi SB} \approx 1$ GeV[1], since perturbative QCD knows nothing about vacuum structure. This expectation has been recently been substantiated by Cohen and Glozman who use quark hadron duality and the operator product expansion to argue that chiral symmetry should be restored high in the baryon spectrum[2]. And in fact, experimental indications of parity restoration have been reported for many years in the baryon spectrum[3].

In the following it will be argued that parity restoration is a natural consequence of quark interactions which are chirally symmetric and relativistic. Of course QCD obeys both of these constraints. Unfortunately, the majority of current quark models fail to satisfy these criteria and they must therefore fail to accurately reproduce the highly excited hadron spectrum.

II. CHIRAL SYMMETRY RESTORATION IN THE HADRON SPECTRUM

As will become clear, examining the restoration of chiral symmetry requires a relativistic formalism. It is therefore natural to explore the hadron spectrum with the aid of interpolating fields which are constructed in the helicity basis (the reader is reminded that the helicity basis is fully relativistic[4]). Meson interpolating states (I concentrate on mesons in the following, however similar arguments apply to baryons) may be written in the form

$$|JM;\lambda\lambda'\rangle \sim \int d\Omega D^{(J)*}_{M,\lambda-\lambda'} b^\dagger_{\mathbf{k},\lambda} d^\dagger_{\mathbf{k},\lambda'}|0\rangle. \tag{1}$$

Phase and normalization factors and the radial wavefunction are left unwritten since they are irrelevant to the following discussion. Note that these interpolating fields have zero overlap with quantum number exotic mesons and hence are only useful in the description of 'canonical' mesons. Finally, (possible) chiral and $U_A(1)$ symmetries impose a definite isospin structure on low energy QCD. This important point will be touched upon again in the conclusions. In the following, perfect isospin symmetry will be assumed and hence all isospin dependence will be neglected.

Parity and charge conjugation relationships are given by

$$P|JM;\lambda\lambda'\rangle = (-)^J|JM;-\lambda-\lambda'\rangle \tag{2}$$

and

$$C|JM;\lambda\lambda'\rangle = (-)^J|JM;\lambda'\lambda\rangle. \tag{3}$$

Thus the complete canonical meson spectrum is spanned by the following interpolating states:

$$|J^{(J+1)(J)}\rangle = \frac{1}{\sqrt{2}}\left(|JM;{+}{+}\rangle - |JM;{-}{-}\rangle\right) \tag{4}$$

$$|J^{(J+1)(J+1)}\rangle = \frac{1}{\sqrt{2}}\left(|JM;{+}{-}\rangle - |JM;{-}{+}\rangle\right) \tag{5}$$

and

$$|J_1^{(J)(J)}\rangle = \frac{1}{\sqrt{2}}\left(|JM;{+}{+}\rangle + |JM;{-}{-}\rangle\right) \tag{6}$$

$$|J_2^{(J)(J)}\rangle = \frac{1}{\sqrt{2}}\left(|JM;{+}{-}\rangle + |JM;{-}{+}\rangle\right). \tag{7}$$

The left hand side of these relations refer to J^{PC} quantum numbers where $(J) \equiv (-)^J$.

It is useful to define temporal correlation functions as follows (T may be simply taken as an arbitrary parameter):

$$C(J^{(J+1)(J)};T) = \langle JM;++|\mathrm{e}^{-HT}|JM;{+}{+}\rangle - \langle JM;++|\mathrm{e}^{-HT}|JM;{-}{-}\rangle \tag{8}$$

$$C(J^{(J+1)(J+1)};T) = \langle JM;+-|\mathrm{e}^{-HT}|JM;{+}{-}\rangle - \langle JM;-+|\mathrm{e}^{-HT}|JM;{+}{-}\rangle. \tag{9}$$

In the $J^{(J)(J)}$ channel one must diagonalize the correlator matrix given by

$$C_{ij}(T) = \langle J_i^{(J)(J)}|\mathrm{e}^{-HT}|J_j^{(J)(J)}\rangle \tag{10}$$

The Hamiltonian is understood to be that of QCD in a convenient gauge. Note that the implicit functional integral may have a nontrivial metric and that the region of integration may be similarly constrained. However, none of these technical issues are relevant to the discussion.

The occurrence of chiral symmetry breaking has several effects on the behaviour of QCD: (i) a nonzero condensate appears, (ii) Goldstone modes are generated, (iii) a dynamical quark mass is generated. Of course these phenomena are all related by the underlying strong QCD dynamics. Thus, for example, Goldstone bosons are fluctuations in the order parameter and are bound states of quasiparticle excitations. Furthermore, these quasiparticle excitations are manifested in QCD via a quark field with a dynamical quark mass, $\mu(p)$, which implicitly defines the nontrivial broken vacuum:

$$\psi(p;m_q = 0) \to \psi(p;m_q = \mu(p)). \tag{11}$$

The classic work of Nambu and Jona-Lasinio[5] gives an explicit example of the quasiparticle (constituent quark) basis embodied in Eq. 11. Similarly, the dynamical quark mass has been related to the chiral condensate by Politzer[6] who used the operator product expansion to obtain

$$\mu(p) \sim \frac{\sigma}{p^2}\langle\bar{\psi}\psi\rangle,\ p \to \infty \tag{12}$$

where $\sigma = 16\pi\alpha_s(p)$ modulo logarithms. This behaviour is very general and is seen in all models of chiral symmetry breaking. Furthermore Politzer showed that the general result

$$\frac{\mu_f(p)}{\mu_{f'}(p)} \to \frac{m_f}{m_{f'}}, \text{as } p \to \infty, \tag{13}$$

holds and that this relationship is true with or without spontaneous chiral symmetry breaking. Here m_f is the bare mass of a quark of flavour f. Setting $f = u$ or d and $f' = b$ in Eq. 13 then demonstrates that $\mu_{u/d}(p) \to 0$ as the momentum gets large (the chiral limit is assumed throughout). Of course the same conclusion follows from Eq. 12.

The central point is that the dominant manifestation of spontaneous chiral symmetry breaking high in the hadronic spectrum is in the quark field expansion and that the effective mass appearing in the quark field must approach zero as the average momentum probed by the quark gets large. But this implies that helicity flip transitions are suppressed at momenta which are large compared to the chiral symmetry breaking scale. (They are also suppressed for massive quarks when the momenta are much larger than the quark masses.) Thus helicity states become good eigenstates (mixing in the $J^{(J)(J)}$ channel is suppressed) and one obtains

$$C(J^{(J+1)(J)}; T) = C_{11}(J^{(J)(J)}; T); \quad \langle p \rangle >> \Lambda_{\chi SB} \tag{14}$$

and

$$C(J^{(J+1)(J+1)}; T) = C_{22}(J^{(J)(J)}; T); \quad \langle p \rangle >> \Lambda_{\chi SB}; \quad J > 0. \tag{15}$$

Finally, the fact that the correlator is an analytic function in T then implies that meson masses obey the same relationships. Thus parity symmetry is restored for highly excited canonical mesons. Furthermore a $J^{\pm\pm}$ doublet structure is expected.

At first sight this conclusion may appear implausible since Fock sector mixing such as generated by gluon exchange or meson loops differ depending on quantum numbers. Nevertheless, if these effects are generated by chirally symmetric interactions (and they are) then the argument given above shows that their effects must become identical in accord with Eqs. 14,15 at high momentum scales. Any interaction, potential or non-potential, local or nonlocal, that is generated by chirally symmetric local interactions (as is the case in massless QCD) must obey the same general constraints laid out here.

A further symmetry occurs in the nonrelativistic limit or at high spin, namely

$$M(J^{(J+1)(J)}) = M(J^{(J+1)(J+1)}). \tag{16}$$

This is the familiar statement that spin decouples from dynamics in the heavy quark limit or when the meson wavefunction is suppressed at the origin. The last three relationships imply that *the conventional meson spectrum falls into degenerate quadruplets* in the high spin/excitation energy limit.

Since nonrelativistic quark models are based on a different momentum regime than that considered here (namely $p << m_q$) it is perhaps not surprising that they can not obtain parity symmetry in the excited spectrum. Such models typically conserve spin and angular momentum separately (with possible perturbative mixing). Thus states of opposite parity differ by partial wave and belong to different multiplets. However, in the relativistic case it is a specific and nonperturbative combination of J-1 and J+1 waves which allows the degeneracy to occur[14].

It is instructive to estimate precisely where in the spectrum parity is restored. Such an estimate is permitted by the assertion that the dominant manifestation of chiral symmetry breaking high in the spectrum is through the dynamical quark mass. In the nearly degenerate limit, mass differences may be estimated perturbatively as $\delta E \sim \langle \lambda\lambda' | \int \bar\psi \Gamma \psi K \bar\psi \Gamma \psi | \mu\mu' \rangle$ where Γ is a Dirac matrix, K is some interaction kernel, and λ, λ', μ, μ' are helicity labels. In the case of helicity flip transition one finds that quark spinors occur in pairs and hence a one percent deviation in meson masses implies that $\langle p \rangle \approx 10 m_q$. For heavy quarks such momenta are experimentally impractical. For light quarks Politzer's result (Eq. 12) may be used to obtain the estimated parity restoration energy

$$E_{restore} \approx -2(10\sigma\langle\bar\psi\psi\rangle)^{1/3} \approx 2.5 \text{ GeV}. \tag{17}$$

It has been noted that the perturbative regime in which chiral symmetry breaking is irrelevant may be so high in energy that well-defined resonances may not exist[2]. It is therefore satisfying that a relatively low mass scale, for which mesons remain easily identifiable, emerges from this analysis.

Unfortunately, little is known of the highly excited meson spectrum. The only isovector states which permit comparison in the excitation spectrum are the scalars and pseudoscalars. These may be compared by computing the mass ratio $(0^{++} - 0^{-+})/(0^{++} + 0^{-+})$. The results are 75%, 6.0%, -2.5%, -1.0%, and -0.5%(-5.8%) for the ground state through the fourth excited states respectively[15]. It is clear that relative mass differences become quite small once masses of roughly 2 GeV are reached.

The only other current option for testing parity doubling is in low lying high angular momentum states. Again, data is sparse but modest progress can be made with $J = 3$ and $J = 4$ states. The RPP[7] lists the states $\rho_3(1690)$ and $\rho_3(1990)$. Equations 14 and 15 lead one to conclude (assuming $E \sim 1700$ MeV is 'large') that a 3^{++} state should exist at approximately 1700 MeV with a 3^{+-} state at roughly 2000 MeV. Indeed, there has been a recent report of a 3^{+-} state at 2032 MeV from Crystal Barrel[8]. The close agreement with expectations lends hope that the predicted mass of the $3^{++}(1700)$ is reasonably accurate. Note that nonrelativistic quark models typically predict that the 3^{++} state is nearly degenerate with the 3^{+-}[10], since they are related by a spin flip. Thus the discovery of a 3^{++} meson with a mass of approximately 1700 MeV would be a dramatic confirmation of the importance of chiral dynamics in QCD and an indication of the limited range of validity of the nonrelativistic approach. Of course, similar statements apply to higher spin states. For example, recent analyses of Crystal Barrel data[9] report isoscalar states as follows: $4^{++}(2018)$, $4^{++}(2283)$, and $4^{-+}(2328)$. The near degeneracy between the higher 4^{++} state and the 4^{-+} is in agreement with Eq. 14 and leads one to expect an isoscalar 4^{--} state at roughly 2000 MeV. Similarly, isovector states are[9] $4^{++}(2005)$, $4^{++}(2255)$, and $4^{-+}(2250)$. Again, an isovector 4^{--} is predicted at 2000 MeV.

A final application of the results presented here concerns the Regge trajectories of hadronic phenomenology. These are traditionally assumed to be linear in the mass squared of the hadrons: $M^2 = \alpha' J + J_0$. However, Eqs. 14 and 15 imply that *Regge trajectories must become nonlinear at moderate spin.* For example the ρ and b_1 trajectories, which may start parallel, must merge past a restoration scale of roughly $M^2 > 6$ GeV^2, or $J > 5$ since they fall in sequences $(J \text{ odd})^{--}$ and $(J \text{ odd})^{+-}$. Experimentally the gap between the trajectories is 0.93 GeV^2 ($J = 1$), 1.27 GeV^2 ($J = 3$), and roughly 0.73 GeV^2 ($J = 5$). Thus there is an indication that the two trajectories are indeed merging. Note that this example is not a good one because the lowest lying ρ_J states have been used to define the ρ trajectory. These may be identified with the helicity states $J_1^{(J)(J)}$, which implies that the comparison is better made with the a_1 trajectory. Thus, this test is actually of the stronger quadruplet degeneracy expected at high J, rather than the weaker degeneracies of Eqs. 14 and 15. Hopefully, meson spectroscopy will advance to the point that all low lying mesons up to $J = 6$ will discovered. Only then can the systematics of chiral restoration be explored with some degree of certainty.

III. CONCLUSIONS

I have argued that the dominant feature of spontaneous chiral symmetry breaking high in the hadronic spectrum is an effective momentum dependent quark mass present in the spinors of the quark field. This effective mass serves to redefine the vacuum and provides and an efficient description of the broken phase of QCD. Furthermore, the effective mass will approach zero as the average momentum probed by the quark gets large. In this case, helicity conservation by chirally symmetric interactions guarantees that two new degeneracies appear in the meson spectrum, namely parity and parity-charge conjugation doublets emerge. Furthermore, at moderately high total angular momentum, these doublets merge into a quadruplet structure, even for low lying states. Finally, the linear Regge trajectories of hadronic phenomenology must be regarded as approximations which are only valid low in the spectrum.

Equations 14 and 15 are consistent with the restoration of $\mathrm{SU}(2)_V \times \mathrm{SU}(2)_A$, and possibly $\mathrm{U}(1)_A \times \mathrm{U}(1)_V$, symmetry. Determining which requires reinstating isospin indices in the formalism. Doing so reveals, for example, that the 1^{++} state becomes degenerate with the 1^{--} due to restored chiral symmetry. Note, however, that it has been argued[2] that the same condensates drive spontaneous SU(2) and U(1) symmetry breaking so that both symmetries are expected to be restored concurrently (if one can neglect the anomalous breaking of U(1) symmetry high in the spectrum). Thus distinguishing these symmetry restoration mechanisms is not necessary. In conclusion, the results presented here are in complete agreement of those of Ref. [2]. What is new is the explicit connection to the constituent quark model, the prediction of quadruplet degeneracy, and the predictions of the previous section.

It is clear that attempts to model both the low lying and the highly excited hadron spectrum must be carried out carefully. The simplest requirement is that any such model be relativistic, otherwise the transition to the parity symmetric region can never occur. Furthermore, model quark interactions should be chirally invariant. Many models employ a scalar confinement potential. We now know that this can only be regarded as an effective low energy interaction and that it cannot correctly describe the highly excited spectrum. A similar example is provided by the phenomenological '$^3P_0$' strong decay model. The model vertex is sometimes written in the relativistic notation $V = g \int \bar{\psi}\psi$. Again, such a chirally noninvariant interaction cannot yield correct high excitations and must be regarded as an effective interaction for low lying states only.

The challenge is to construct new models which are chirally invariant, relativistic, and which still reproduce the successes of earlier models in the low lying spectrum. This need not be an insurmountable problem, for example a model for mesons which incorporates these features and explicitly demonstrates the effects discussed here is developed in Ref. [11]. Another possible resolution is explored in Ref. [12]. How a new strong decay model may be constructed which obeys these constraints is discussed in Section V.C of Ref. [13].

Acknowledgments

I am grateful to Leonid Glozman, Bob Jaffe, and John Terning for fruitful discussions and to Leonid Glozman for directing my attention to Refs. [9] and the latter three of Refs. [2]. This work was supported by the DOE under contracts DE-FG02-00ER41135 and DE-AC05-84ER40150.

[1] A. Le Yaouanc, L. Oliver, S. Ono, O. Pène and J. C. Raynal, Phys. Rev. D **31**, 137 (1985), see especially the discussion after Eq. 7.15.
[2] T. D. Cohen and L. Y. Glozman, Phys. Rev. D **65**, 016006 (2002). See also L.Y. Glozman, Phys. Lett. B **539**, 257 (2002) where parity restoration in the meson spectrum is discussed; Phys. Lett. B **541**, 115 (2002); and arXiv:hep-ph/0304087.
[3] B. Nefkens, piN Newsletter **11**, 145 (1995); see also the discussion in S. Capstick *et al.*, "Key Issues in Hadronic Physics", arXiv:hep-ph/0012238.
[4] A.D. Martin and T.D. Spearman, "Elementary Particle Theory", (Amsterdam, North-Holland Publishing Company, 1970).
[5] Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961).
[6] D. Politzer, Nucl. Phys. **B117**, 397 (1976).
[7] "Review of Particle Properties", K. Hagiwara *et al.*, Phys. Rev. **D66**, 1 (2002).
[8] A. V. Anisovich *et al.*, Phys. Lett. B **542**, 8 (2002).
[9] A. V. Anisovich *et al.*, Phys. Lett. B **449**, 154 (1999); Phys. Lett. B **491**, 47 (2000); Phys. Lett. B **517**, 261 (2001).
[10] S. Godfrey and N. Isgur, Phys. Rev. D **32**, 189 (1985).
[11] N. Ligterink and E. S. Swanson, "A Coulomb gauge model of mesons", arXiv:hep-ph/0310070, to appear Phys. Rev. **C**.
[12] P. Bicudo and G.M. Marques, "Chiral Symmetry Breaking and Scalar Confinement", arXiv:hep-ph/0309231.
[13] A. P. Szczepaniak and E. S. Swanson, Phys. Rev. D **65**, 025012 (2002).
[14] Specifically, $|1\rangle_J = |(J-1)\rangle/\sqrt{3} - \sqrt{2}|(J+1)\rangle/\sqrt{3}$ and $|2\rangle_J = \sqrt{2}|(J-1)\rangle/\sqrt{3} + |(J+1)\rangle/\sqrt{3}$.
[15] The following masses from the RPP[7] have been employed: $\pi(138)$, $a_0(980)$, $\pi(1300)$, $a_0(1474)$, $\pi(1800)$. I have used the $f_J(1713)$ mass as an estimate of the $a_0(3S)$ mass. Note, however, that Ref.[9] claim an f_0 at 1770 MeV. Further states are obtained from Ref. [9]: $\pi(2070)$, $\pi(2360)$, $a_0(\sim 2025)/f_0(2040)$, and the $f_0(2337)$. These references also report an f_0 at 2102 MeV, which provides the last ratio in brackets above.